Obliczyć:

$$\int_{]-\infty,+\infty[} f(x)dx \quad \text{jeżeli:}$$

$$f(x)=\begin{cases} \dfrac{-\sin(x)+x\cos(x)}{x^2} & dla \quad x\in]-\infty,0[\\ \dfrac{x}{\sqrt[3]{2x-1}} & dla \quad x\in[0,1] \\ \dfrac{1}{\sqrt{(1+x^2)\ln^3(x+\sqrt{x^2+1})}} & dla \quad x\in]1,+\infty[\end{cases}$$

Rozwiązanie:

❶

$$I_1=\int_{]-\infty,0[}\frac{-\sin(x)+x\cos(x)}{x^2}dx=\lim_{\substack{A\to-\infty\\ B\to 0}}\int_A^B\frac{-\sin(x)+x\cos(x)}{x^2}dx$$

❷

W $\dfrac{x}{\sqrt[3]{2x-1}}$ $x\neq 0.5$, więc $\dfrac{x}{\sqrt[3]{2x-1}}$ dzielimy na 2 części

$$I_2=\int_{[0,0.5[}\frac{x}{\sqrt[3]{2x-1}}dx+\int_{]0.5,1]}\frac{x}{\sqrt[3]{2x-1}}dx=\lim_{A\to 0.5}\int_0^A\frac{x}{\sqrt[3]{2x-1}}dx+\lim_{B\to 0.5}\int_B^1\frac{x}{\sqrt[3]{2x-1}}dx$$

❸

$$I_3=\Phi(1)+\int_{]1,\infty[}\frac{1}{\sqrt{(1+x^2)\ln^3(x+\sqrt{x^2+1})}}dx=\lim_{\substack{A\to 1\\ B\to\infty}}\int_A^B\frac{1}{\sqrt{(1+x^2)\ln^3(x+\sqrt{x^2+1})}}dx$$

Liczymy całki:

①

$$\int\frac{-\sin(x)+x\cos(x)}{x^2}dx=\ldots=\frac{\sin(x)}{x}+C$$

②

$$\int\frac{x}{\sqrt[3]{2x-1}}dx=\ldots=\frac{3}{4}\sqrt[3]{(2x-1)^2}\left(\frac{2}{5}x+\frac{3}{10}\right)+C$$

③

$$\int\frac{1}{\sqrt{(1+x^2)\ln^3(x+\sqrt{x^2+1})}}dx=\ldots=-\frac{2}{\sqrt{\ln(x+\sqrt{x^2+1})}}+C$$

❶

$$I_1 = \lim_{\substack{A\to-\infty\\ B\to 0}} \int_A^B \frac{-\sin(x)+x\cos(x)}{x^2}dx = \lim_{\substack{A\to-\infty\\ B\to 0}} \left[\frac{\sin(x)}{x}\right]_A^B = \lim_{\substack{A\to-\infty\\ B\to 0}} \left(\frac{\sin(B)}{B} - \frac{\sin(A)}{A}\right) = 1-0=1$$

❷

$$I_2 = \lim_{A\to 0.5} \int_0^A \frac{x}{\sqrt[3]{2x-1}}dx + \lim_{B\to 0.5} \int_B^1 \frac{x}{\sqrt[3]{2x-1}}dx = \lim_{A\to 0.5}\left[\frac{3}{4}\sqrt[3]{(2x-1)^2}\,(\frac{2}{5}x+\frac{3}{10})\right]_0^A +$$

$$\lim_{B\to 0.5}\left[\frac{3}{4}\sqrt[3]{(2x-1)^2}\,(\frac{2}{5}x+\frac{3}{10})\right]_B^1 = \lim_{A\to 0.5}\left(\left[\frac{3}{4}\sqrt[3]{(2A-1)^2}\,(\frac{2}{5}A+\frac{3}{10})\right]-\left[\frac{3}{4}\sqrt[3]{(2*0-1)^2}\,(\frac{2}{5}*0+\frac{3}{10})\right]\right)+$$

$$\lim_{B\to 0.5}\left(\left[\frac{3}{4}\sqrt[3]{(2*1-1)^2}\,(\frac{2}{5}*1+\frac{3}{10})\right]-\left[\frac{3}{4}\sqrt[3]{(2B-1)^2}\,(\frac{2}{5}B+\frac{3}{10})\right]\right)=$$

$$\left(0-\frac{9}{40}\right)+\left(\left[\frac{3}{4}(\frac{4}{10}+\frac{3}{10})\right]-0\right) = -\frac{9}{40}+\frac{21}{40}=\frac{12}{40}=\frac{3}{10}$$

❸

$$I_3 = \lim_{\substack{A\to 1\\ B\to\infty}} \int_A^B \frac{1}{\sqrt{(1+x^2)\ln^3(x+\sqrt{x^2+1})}}dx = \lim_{\substack{A\to 1\\ B\to\infty}}\left[-\frac{2}{\sqrt{\ln(x+\sqrt{x^2+1})}}\right]_A^B =$$

$$\lim_{\substack{A\to 1\\ B\to\infty}}\left(-\frac{2}{\sqrt{\ln(B+\sqrt{B^2+1})}}+\frac{2}{\sqrt{\ln(A+\sqrt{A^2+1})}}\right)=$$

$$\left(0+\frac{2}{\sqrt{\ln(1+\sqrt{2})}}\right)=\frac{2}{\sqrt{\ln(1+\sqrt{2})}}$$

$$I = I_1 + I_2 + I_3$$

$$I = 1+\frac{3}{10}+\frac{2}{\sqrt{\ln(1+\sqrt{2})}}$$