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Zestaw IV

Zadanie 2

Obliczenia całek nieoznaczonych:

$$I_1 = \int \frac{e^x}{e^{2x}+3}dx = \begin{cases} e^x = \sqrt{3}t \\ e^x dx = \sqrt{3}dt \end{cases} = \int \frac{\sqrt{3}}{3t^2+3}dt = \frac{\sqrt{3}}{3}\int \frac{1}{t^2+1}dt = \frac{\sqrt{3}}{3} arctgt + C = \frac{\sqrt{3}}{3} arctg \frac{e^x}{\sqrt{3}} + C$$

$$I_2 = \int \frac{x^2}{\sqrt{9-x^2}}dx = (Ax+B)\sqrt{9-x^2} + \lambda \int \frac{dx}{\sqrt{9-x^2}} = \# \quad / \, ( )`$$

$$\frac{x^2}{\sqrt{9-x^2}} = A\sqrt{9-x^2} - (Ax+B)\frac{x}{\sqrt{9-x^2}} + \frac{\lambda}{\sqrt{9-x^2}} \quad / \sqrt{9-x^2}$$

$x^2 = 9A - Ax^2 - Ax^2 - Bx + \lambda$ a zatem

$$A = -\frac{1}{2}$$
$$B = 0$$
$$\lambda = \frac{9}{2}$$

$$\# = -\frac{x}{2}\sqrt{9-x^2} + \frac{9}{2}\int \frac{dx}{\sqrt{9-x^2}} = @$$

$$\int \frac{dx}{\sqrt{9-x^2}} = \begin{cases} x = 3t \\ dx = 3dt \end{cases} = 3\int \frac{dt}{3\sqrt{1-t^2}} = \arcsin t + C = \arcsin \frac{x}{3} + C$$

$$@ = -\frac{x}{2}\sqrt{9-x^2} + \frac{9}{2}\arcsin \frac{x}{3} + C$$

$$I_3 = \int \frac{x^2}{(x-1)^{10}}dx = \left\{ \begin{array}{l} x-1=t \\ dx = dt \end{array} \right\} = \int \frac{t^2+2t+1}{t^{10}}dt = \int \frac{1}{t^8}dt + \int \frac{2}{t^9}dt + \int \frac{1}{t^{10}}dt =$$

$$-\frac{1}{7}t^{-7} - \frac{1}{4}t^{-8} - \frac{1}{9}t^{-7} + C =$$

$$-(x-1)^{-7}\left[\ \frac{1}{7} + \frac{1}{4}(x-1)^{-1} + \frac{1}{9}(x-1)^{-2}\ \right] + C = -\frac{36x^2-9x+1}{252(x-1)^9} + C$$

$$I_4 = \int \frac{25}{4^{10}}dx = \frac{25}{4^{10}}x + C$$

$$\Phi(x) = \begin{cases} \underset{]-\infty,x]}{I_1} \text{ dla } x \in]-\infty,0] \\ \Phi(0) + \underset{[0,x]}{I_2} \text{ dla } x \in]0,3[\\ \lim_{c\to 3^-}\Phi(c) + \underset{[3,x]}{I_3} = \Phi(0) + \underset{[0,3[}{I_2} + \underset{[3,x]}{I_3} \text{ dla } x \in [3,5] \\ \Phi(5) + \underset{[5,x]}{I_4} = \Phi(0) + \underset{[0,3[}{I_2} + \underset{[3,5]}{I_3} + \underset{[5,x]}{I_4} \text{ dla } x \in]5,+\infty[\end{cases}$$

Obliczenia pomocnicze:

$$\underset{]-\infty,x]}{I_1} = \lim_{A\to-\infty}\left[\frac{\sqrt{3}}{3}arctg\frac{e^x}{\sqrt{3}}\right]_A^x = \frac{\sqrt{3}}{3}arctg\frac{e^x}{\sqrt{3}} - \lim_{A\to-\infty}\frac{\sqrt{3}}{3}arctg\frac{e^A}{\sqrt{3}} = \frac{\sqrt{3}}{3}arctg\frac{e^x}{\sqrt{3}}$$

$$\Phi(0) = \frac{\sqrt{3}}{3}arctg\ \frac{e^0}{\sqrt{3}} = \frac{\sqrt{3}}{3}\,\frac{\pi}{6} = \frac{\sqrt{3}\pi}{18}$$

$$\underset{[0,x]}{I_2} = \left[-\frac{x}{2}\sqrt{9-x^2} + \frac{9}{2}\arcsin\frac{x}{3}\right]_0^x = -\frac{x}{2}\sqrt{9-x^2} + \frac{9}{2}\arcsin\frac{x}{3} - \left[0 + \frac{9}{2}\arcsin 0\right] =$$

$$-\frac{x}{2}\sqrt{9-x^2} + \frac{9}{2}\arcsin\frac{x}{3}$$

$$\underset{[0,3[}{I_2} = \lim_{B\to 3^-}\left[-\frac{x}{2}\sqrt{9-x^2} + \frac{9}{2}\arcsin\frac{x}{3}\right]_0^B = \lim_{B\to 3^-}\left[-\frac{B}{2}\sqrt{9-B^2} + \frac{9}{2}\arcsin\frac{B}{3}\right] - 0 = \frac{9\pi}{4}$$

$$\underset{[3,x]}{I_3} = \left[-\frac{36x^2 - 9x + 1}{252(x-1)^9} \right]_3^x = -\frac{36x^2 - 9x + 1}{252(x-1)^9} + \frac{36*9 - 27 + 1}{252(3-1)^9} = -\frac{36x^2 - 9x + 1}{252(x-1)^9} + \frac{149}{63*2^{10}}$$

$$\underset{[3,5]}{I_3} = \left[-\frac{36x^2 - 9x + 1}{252(x-1)^9} \right]_3^5 = -\frac{36*25 - 45 + 1}{252(5-1)^9} + \frac{149}{63*2^{10}} = \frac{18965}{63*2^{17}}$$

$$\underset{[5,x]}{I_4} = \left[\frac{25}{4^{10}} x \right]_5^x = \frac{25}{4^{10}} x - \frac{125}{4^{10}}$$

$$\Phi(x) = \begin{cases} \underset{]-\infty,x]}{I_1} \text{ dla } x \in]-\infty,0] \\ \Phi(0) + \underset{[0,x]}{I_2} \text{ dla } x \in]0,3[\\ \lim_{c \to 3^-} \Phi(c) + \underset{[3,x]}{I_3} = \Phi(0) + \underset{[0,3[}{I_2} + \underset{[3,x]}{I_3} \text{ dla } x \in [3,5] \\ \Phi(5) + \underset{[5,x]}{I_4} = \Phi(0) + \underset{[0,3[}{I_2} + \underset{[3,5]}{I_3} + \underset{[5,x]}{I_4} \text{ dla } x \in]5,+\infty[\end{cases}$$

Odpowiedź:

$$\Phi(x) = \begin{cases} \frac{\sqrt{3}}{3} arctg \frac{e^x}{\sqrt{3}} \quad \text{dla } x \in]-\infty,0] \\ -\frac{x}{2}\sqrt{9 - x^2} + \frac{9}{2}\arcsin\frac{x}{3} + \frac{\sqrt{3}\pi}{18} \quad \text{dla } x \in]0,3[\\ -\frac{36x^2 - 9x + 1}{252(x-1)^9} + \frac{149 + 1792\pi(2\sqrt{3} + 81)}{63*2^{10}} \quad \text{dla } x \in [3,5] \\ \frac{25}{4^{10}} x - \frac{125}{4^{10}} + \frac{\pi(2\sqrt{3} + 81)}{18} + \frac{18965}{63*2^{17}} \text{ dla } x \in]5,+\infty[\end{cases}$$

$$\Phi'(x) = \begin{cases} \dfrac{e^{x}}{e^{2x}+3} & \text{dla x} \in]-\infty,0[\\ \dfrac{x^{2}}{\sqrt{9-x^{2}}} & \text{dla x} \in]0,3[\\ \dfrac{x^{2}}{(x-1)^{10}} & \text{dla x} \in]3,5] \\ \dfrac{25}{4^{10}} & \text{dla x} \in]5,+\infty[\end{cases}$$