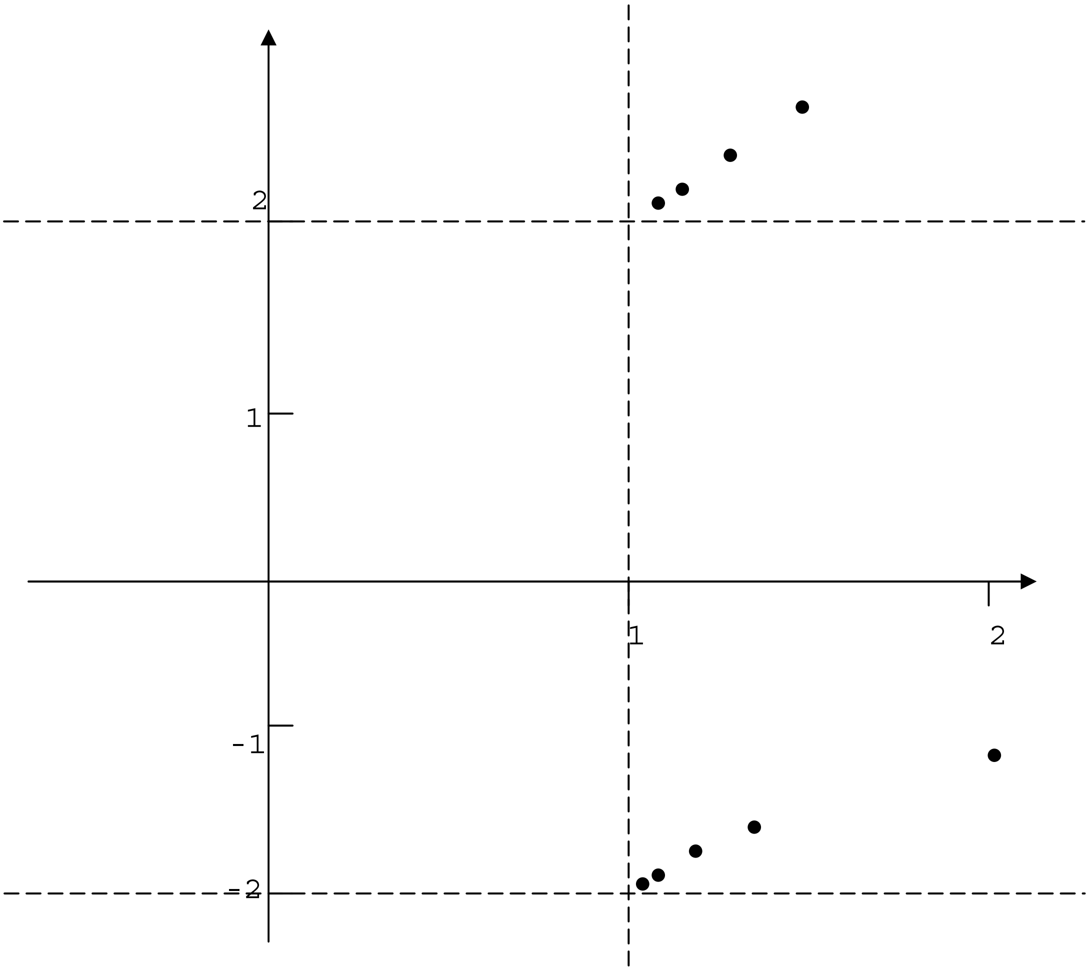
ZESTAW V zad.4 opracował Michał Wdaniec

a) $A=\left\{(1+\frac{1}{n},2(-1)^n+\frac{1}{n});n\in N\right\}$



$\overline{A}=A\cup\{(1,2),(1,-2)\};$

(1,2) jest punktem skupienia A : $\Leftrightarrow \underset{(x_n)_{n\in N}\subset A}{\exists} : \lim_{n\to\infty} x_n = (1,2);$

niech $x_n=(1+\frac{1}{2n},2+\frac{1}{2n})\subset A$, wtedy:

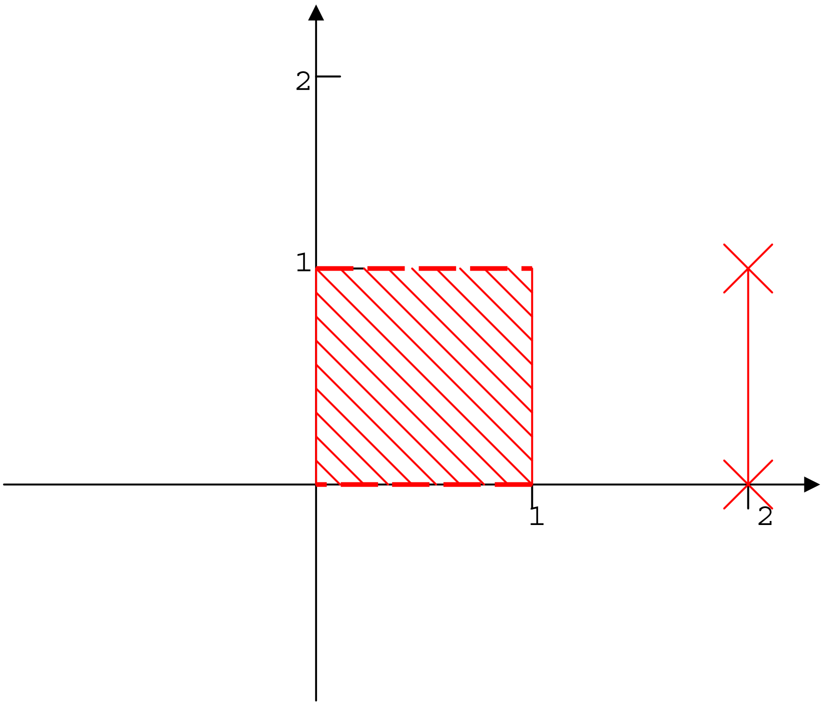
$\lim_{n\to\infty} x_n = \lim_{n\to\infty}(1+\frac{1}{2n},2+\frac{1}{2n})=(1,2)$ c.n.d

Dla punktu (1,-2) analogicznie.

$\partial A=\overline{A}$

b)

$A = ([0,1] \cup \{2\}) \times]0,1[$



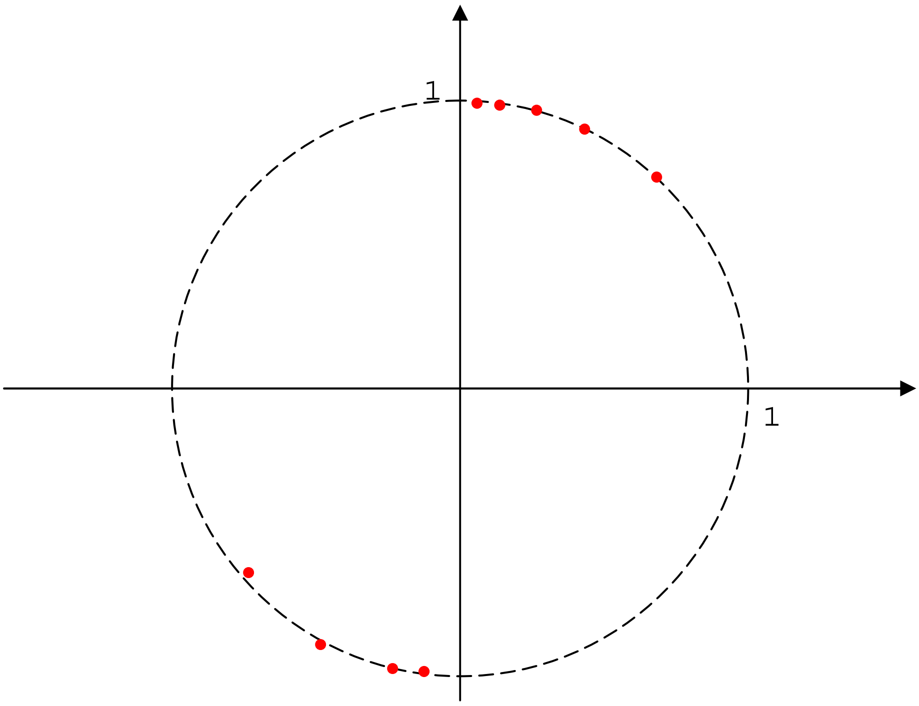
$\overline{A} = A \cup (([0,1] \cup \{2\}) \times \{0,1\})$

$\{\text{punkty skupienia A}\} = (([0,1] \cup \{2\}) \times [0,1])$

$\partial A = (([0,1] \cup 2) \times \{0,1\}) \cup (\{0,1,2\} \times [0,1])$

c)

$$A=\bigcup_{n=1}^{\infty} A_n, gdzie\ A_n=\{(x,y): y=nx \wedge x^2+y^2=1; n\in N\}$$



$\overline{A}=A\cup\{0,1\}$

(0,1) i (0,-1) jest punktem skupienia A

: $\Leftrightarrow \underset{(x_n)_{n\in N}\subset A}{\exists} : \lim_{n\to\infty} x_n=(0,1);$ i $\underset{(y_n)_{n\in N}\subset A}{\exists} : \lim_{n\to\infty} y_n=(0,-1);$

Z rozwiązania równań danych w opisie zbioru mamy:

$x_n=\left(\frac{1}{\sqrt{1+n^2}},\frac{n}{\sqrt{1+n^2}}\right), n\in N \Rightarrow x_n\subset A$ i $y_n=\left(\frac{-1}{\sqrt{1+n^2}},\frac{-n}{\sqrt{1+n^2}}\right), n\in N \Rightarrow y_n\subset A$

$\lim_{n\to\infty} x_n=\lim_{n\to\infty}\left(\frac{1}{\sqrt{1+n^2}},\frac{n}{\sqrt{1+n^2}}\right)=(0,1)$ i $\lim_{n\to\infty} y_n=\lim_{n\to\infty}\left(\frac{-1}{\sqrt{1+n^2}},\frac{-n}{\sqrt{1+n^2}}\right)=(0,-1)$ c.n.d

$\partial A=\overline{A}$